# AN EXPERIMENTAL STUDY OF STEAM CONDENSATION ON A LAMINAR WATER SHEET

# D. HASSON, D. LUSS\* and U. NAVON<sup>†</sup>

Faculty of Chemical Engineering, Technion-Israel Institute of Technology, Haifa, Israel

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Abstract-A method has been developed to study the heat transfer in steam condensation on laminar water sheets generated by the fan spray nozzle. Local heat-transfer coefficients are reliably determined, without probing instruments, from interferometric measurements of the thickness increase due to steam condensation.

It is found that the slight amounts of dissofved air, released during heating of the sheet, considerably increase the resistance to heat transfer. Tests with de-aerated water are used to confirm the analytical solution for the heating of the fan spray sheet without surface resistance, which has been presented in an earlier paper [2].

The effects of steam pressure and air content in steam on the heat-transfer coefficient are investigated. It is found that the heat-transfer coefficient, though very high (100 000 to 200 000 kcal/h  $m^2$  degC), remains unaltered when steam pressure is reduced from above atmospheric to sub-atmospheric pressure. This result implies that interface resistance is negligible in vacuum condensation equipment, contrary to the view expressed by some authors.

The decrease in the heat-transfer coefficient with increasing air content of steam is empirically correlated. A simple design equation is derived which, together with the empirical correlation, can be used for practical calculations of direct contact condensation equipment.

#### **NOMENCLATURE**

- slope of interferometric curve ;  $a_{\rm s}$
- $\overline{A},$ area of one face of sheet:
- $B_{\cdot}$ optical constant;
- specific heat;  $\mathcal{C}_{\bullet}$
- per cent by volume air in steam; C,
- percentage error due to heating effects e. in air analysis;
- constant; F.
- Graetz number; Gz.
- interface coefficient of heat transfer;  $h_i$
- $h_x$ local overall heat-transfer coefficient:
- $h/h_0$ ratio of heat-transfer coefficient with air in steam to the coefficient with practically pure steam;
- $H<sub>s</sub>$ steam enthalpy referred to inlet water temperature as datum;
- mechanical equivalent of heat;  $J_{\star}$
- thickness parameter; k.
- thermal conductivity; K,
- L. latent heat;
- molecular weight ;  $M_{-}$
- fringe number;  $\overline{n}$ .
- local Nusselt number based on hy- $Nu_{x}$ draulic diameter of the sheet;
- $\overline{Nu}_r$ . average  $Nu<sub>x</sub>$  measured for a sheet;
- absolute pressure in air analysing  $P_1$ apparatus before sampling;
- absolute pressure after sampling;  $P_3$
- total pressure difference;  $\Delta P$ .
- pressure difference due to heating effect ;  $\Delta P_{1}$ .
- local heat-transfer coefficient;  $q_x$
- volumetric flow rate;  $Q_{\star}$
- fringe number of first clearly resolved r, fringe;
- R. universal gas constant;
- sheet thickness ;  $S_{\star}$
- $\overline{T}$ . absolute temperature of vapour;
- $T_0$ , inlet water temperature;
- $T_1$ , absolute temperature of collected air sample;
- absolute temperature of hot trapped air;  $T_3,$
- temperature of saturated steam;  $T_{s}$ .
- volumetric average velocity; ū,
- $(u^2)$ <sub>m</sub>, mean of the velocity squared;

<sup>\*</sup> Present address: Department of Chemical Engineering, University of Minnesota, U.S.A.

f Present address : Department of Mechanical Engineering, Princeton University, U.S.A.

- L!, volume of voids in molecular *sieves*  bed *;*
- $V_b$ , volume of air collecting bulb;
- $x$ , radial distance from sheet origin;<br> $X$ , wetness fraction of steam;
- $X$ , wetness fraction of steam;<br> $W$ , weight of adsorbed water.
- weight of adsorbed water.

#### Greek symbols

- a. condensation coefficient ;
- $\beta$ , ratio of velocity squared means;
- $\gamma$ , ratio of velocity means,  $(\beta/\beta_0)^{1/2}$ ;<br>  $\delta$ , angle of light incidence;
- **8,** angle of light incidence;
- $\lambda$ , light wavelength;
- 
- $\mu$ , refractive index;<br> $\phi$ , angle subtended angle subtended by sheet sector;
- *P:* liquid density.

#### Subscripts

- 0, refers to the case of no steam condensation;
- max, denotes values in the saturation region;
- *n*, at the *n*<sup>th</sup> fringe;<br>*r*, at the *r*<sup>th</sup> fringe.
- at the  $r<sup>th</sup>$  fringe.

# **INTRODUCTION**

**BECAUSE** of experimental difficulties. no reliable data have been hitherto available on the heat transfer in steam condensation on water jets. Measurements taken with a probe introduced into the flowing jet are uncertain; a serious error may arise from the flow disturbance caused by the probe.

The method developed in the present investigation overcomes these difficulties by not requiring probe measurements. Local heattransfer coefficients for steam condensation on fan spray water sheets have been determined by interferometric measurement of the increase in sheet thickness due to condensation.

Steam condensation on conical water sheets of swirl spray nozzles was experimentally studied by Weinberg [I]. It was found that the temperature rise of the sheet is very rapid. Most of the heating took place in the film phase of the spray. before break-up of the sheet occurred.

Weinberg's result for the local heat-transfer coefficient are of limited value. They are based on measurements of temperature and velocity taken with inadequately small probes inserted in relatively thin sheets. Moreover, they are

uniustifiably correlated on the basis of the functional form applying to heat transfer in forced convection through pipes.

# **FACTORS INVESTIGATED**

A previous paper [2] describes theoretical solutions for the heating of laminar water sheets by the condensation of saturated steam, with and without an external resistance on the sheet surface. One aspect of this experimental study was the search of appropriate conditions for verifying these theoretical ana!yses. Since the magnitude of the surface resistance on the sheet is unknown, the problem was to ensure condensation without surface resistance, enabling comparison of theory with experiment.

The main efforts of this investigation were directed to the measurement of local heat-transfer coefficients under conditions of practical interest. The principal objectives were to determine whether the heat-transfer coefficient significantly decreases when steam pressure is reduced from above atmospheric to sub-atmospheric pressure and to examine the effect of air content in the steam on the heat-transfer coefficient. The background to these objectives is discussed below.

#### *Condensation at sub-atmospheric pressures*

According to gas kinetics concepts, steam condensation occurs when the number of vapour molecules striking the condensed liquid surface exceeds the number of molecules leaving it. This rate process imposes an "interfacial resistance" which, under most circumstances, is negligibly small compared to other heat- and mass-transfer resistances acting on the system, However, at sufficiently low steam pressures, the interface resistance can become appreciable. The interfacial heat-transfer coefficient diminishes with decreasing vapour temperature and hence, with decreasing vapour pressure. This is seen from the following formula, obtained by combining the Hertz-Knudsen equation with the Clausius-Clapeyron equation [3, 41:

$$
h_i = a \left(\frac{M}{2\pi RT}\right)^{1/2} \frac{J L^2}{T V} \tag{1}
$$

where  $h_i$  is the interface coefficient for heat transfer,  $M$ —the molecular weight,  $R$ —the gas

constant, J—the mechanical equivalent of heat, T-the absolute temperature of the vapour,  $L$ --the latent heat at  $T$  and  $V$ --the specific volume of the vapour at *T.* The condensation coefficient a is an experimentally measured factor which can either be equal to or less than one. It represents the ratio of condensing molecules to the total number of molecules striking the condensing surface as predicted from gas kinetics arguments.

In order to compare the value of the interfacial resistance  $1/h_i$  to the magnitude of other resistances acting on a condensing system, it is necessary to know the value of a. There is, however, considerable uncertainty regarding the value of  $\alpha$  and the factors affecting its magnitude. Conflicting results were obtained by several of the workers who measured the condensation coefficient of water; values of a ranging from 0.036 to 1 have been reported. A comprehensive review of these results is given by Knacke and Stranski [5].

The extreme differences in the value claimed for a bear important practical implications for low-pressure steam condensers. The steam side heat-transfer coefficient in tubular surface condensers is of the order of 10 000 kcal/ hm<sup>2</sup> degC. Using the same units,  $h_i$  calculated from equation (1) is 6 750 000 for  $\alpha = 1$  and 238 000 for  $\alpha = 0.036$ , when condensation takes place at atmospheric pressure (Table I). It is seen that even with the lower value of  $\alpha$ , interfacial resistance is negligibly small compared to the steam side coefficient.

*Table* 1. *Values of the interface heut-transfer coefficient for water assuming*  $a = 0.036$  *and* 1

Absolute pressure,	$h_i$ kcal/h m <sup>2</sup> degC		
(in Hg)	$a=1$	$a = 0.036$	
		Ω	
	500 000	18 000	
2	850 000	30 500	
3	1 220 000	44 000	
4	1490 000	53 500	
5	1800000	65 000	
6	2 100 000	75 700	
30	6 750 000	238 000	

However, for low-pressure condensation, within the limit reached in industrial practice, interfacial resistance may or may not be negligible, depending on the value of  $\alpha$ . For instance, at an absolute pressure of 1 in Hg,  $h_i$  would have the value of 500 000 if a were to equal 1 and 18 000 if a were O-036 (Table 1). In the former case interfacial resistance should be negligibly small compared to the steam side resistance, as assumed by Othmer [6]. In the latter case, interfacial resistance should be of the same order of magnitude as the steam side resistance, as suggested by Silver [3].

The controversial question of the magnitude of surface resistance in low-pressure condensation takes an added significance in direct contact condensers. Very high heat-transfer coefficients are expected in steam condensation on thin water sheets of low conduction resistance. This heat-transfer coefficient would be significantly reduced at a comparatively low vacuum if interface resistance were governed by the lower value of  $\alpha$ , but would hardly vary, if interfacial resistance were to correspond to the higher value of  $\alpha$ . Comparison of the local heattransfer coefficient, measured at pressures below and above atmospheric, can therefore shed light on a question of great practical and theoretical interest.

# *Condensation in the presence of air*

It is well known that the heat-transfer coefficient in the filmwise condensation of steam on a cold solid surface is considerably reduced when the steam is contaminated by slight amounts of air. The heat-transfer coefficient of practically pure steam is reduced to half its value when the steam contains only  $0.5$  per cent air [6, 7].

The quantitative aspects of this problem are little understood and it has so far been only empirically treated [6, 7]. The usual explanation for the deleterious effect of small quantities of air in steam is based on a diffusion mechanism. Air concentration tends to increase in the vapour layer adjacent to the condensing surface since, in the phase transition, air carried with the steam is rejected near the surface. A steady state is reached when the backward diffusion of the air against the flow of the steam is balanced by the rate at which air is carried towards the condensing

surface. The thermal resistance of the increased air concentration layer near the surface is then responsible for the decrease in the heat-transfer coefficient.

A different mechanism was proposed by Baer and McKelvey [4] in their experimental investigation of the filmwise condensation of methanol vapour containing air on a solid surface. The reduction in the heat-transfer coefficient with increasing air concentration was analysed assuming that, at a given vapour pressure, the condensation coefficient a is not constant but decreases with air concentration.

The justification given by Baer and McKelvey for rejecting the accepted diffusion mechanism as the explanation of their results rests on an ambiguous negative proof, open to doubt.

The effect of air concentration on the heattransfer coefficient in direct contact condensation has not been examined in previous investigations. Evidently, air concentration. studied in the present work, is expected to be one of the most significant variables.

#### **THEORY OF THE EXPERIMENTAL METHOD**

The essence of the interferometric technique, used to measure local heat-transfer coefficients of fan spray water sheets exposed to steam, lies in the possibility of determining the thickness increase resulting from steam condensation. The growth rate of this thickness increase is directly related to the condensation rate and hence, to the local heat-transfer coefficient.

The following derivations aim to relate the interferometrically determined thickness measurements with the local heat-transfer coefficient.

# *Flow in the sheet*

Figure 1 shows the essential features of the fan spray sheet [S]. The streamlines of the sheet travel at a constant velocity along radial lines originating from a hypothetical point situated behind the orifice. Ideally, the thickness of the sheet is uniform along a circumferential line except near the thickened edges of the curved boundaries. All derivations given below are therefore based on a narrow sector subtending an angle of  $\phi$  radians in the central portion of the sheet, far removed from the boundaries (Fig. 1).



FIG. 1. Fan spray sheet geometry.

When a water sheet flows without steam condensation, the volumetric flow rate,  $Q_0$ , remains constant within the sector of angle  $\phi$ . The area normal to the flow at a distance  $x$ from the origin is the product of the arc  $\phi x$ and the thickness  $s_0$ . Hence,

$$
Q_0 = \phi \times s_0 \overline{u}_0 = \phi \cdot k_0 \overline{u}_0 \tag{2}
$$

where  $\bar{u}_0$  is the average velocity across the transverse section. The thickness parameter,  $k_0$ , is a constant depending only on nozzle geometry, defined by:

$$
k_0 = x s_0 \tag{3}
$$

The subscript 0 is used to refer to the case of constant flow rate with no steam condensation.

When steam condensation occurs on the same sheet, the thickness s at a given distance  $x$  is increased. For this case, equation (2) takes the form :

$$
Q = \phi \times s \bar{u} = \phi \times \bar{u} \tag{4}
$$

where

$$
k = x s \tag{5}
$$

Here  $Q$ ,  $k$  and  $\bar{u}$  are no longer constants but vary with  $x$  depending on the condensation rate.

From equations (2) and (4),

$$
\frac{\bar{u}}{\bar{u}_0} = \left(\frac{Q}{Q_0}\right) \left(\frac{k_0}{k}\right) \tag{6}
$$

The velocity ratio can be independently related to the thickness parameter using the momentum equation. Momentum in the  $x$  direction only is considered since with symmetrical condensation on opposite sides of the sheet, other components cancel each other. The condensate joining the sheet originates from steam which, initially, was virtually stagnant. Hence the exit momentum of the water sheet must be conserved along the  $x$ direction, so that:

$$
\rho \phi k_0 (u_0^2)_m = \rho \phi k (u^2)_m \tag{7}
$$

where each of the velocity terms is a mean of the squared quantity.

Let

$$
\beta = \frac{\bar{u}^2}{(u^2)_m}.\tag{8}
$$

Using equations (6), (7) and (8), flow rate increase due to condensation at the distance  $x$ is related to the thickness increase and to the velocity decrease by :

$$
\frac{Q}{Q_0} = \left(\frac{\beta}{\beta_0}\right)^{1/2} \left(\frac{k}{k_0}\right)^{1/2} = \left(\frac{\beta}{\beta_0}\right) \left(\frac{\bar{u}_0}{\bar{u}}\right). \tag{9}
$$

The exact velocity profile in the transverse direction is not known. It will be assumed that this velocity profile is the same with and without steam condensation, for each transverse section of the sheet. This assumption is not considered to lead into a significant error since the velocity profile in a jet is expected to approach that of plug flow. According to this assumption:

$$
\gamma = \left(\frac{\beta}{\beta_0}\right)^{1/2} = 1. \tag{10}
$$

Experimental evidence supporting the above assumption is given later.

# *Temperature variation in the sheet*

The temperature variation can be followed from the thickness increase due to condensation by making a heat balance. Since the exact condition of the steam near the condensing surface is unknown, the derivation will be made general by considering steam of unknown quality.

If  $X$  is the wetness fraction of the steam, the enthalphy  $H_s$  of steam at the temperature  $T_s$ , referred to the inlet sheet temperature  $T_0$ , is given by:

$$
H_s = (1 - X)L + c (T_s - T_0). \qquad (11)
$$

Taking  $T_0$  as the datum temperature, an enthalpy balance equating heat input due to the condensing steam over the distance  $x$  with heat output at  $x$  (Fig. 2) shows that:

$$
\rho(Q - Q_0) H_s = \rho Q c(\overline{T} - T_0) \qquad (12)
$$

where  $\bar{T}$  is the average temperature in the transverse section of the sheet defined by:

$$
\bar{T} = \int_{0}^{s/2} u \, T \, dy / \int_{0}^{s/2} u \, dy. \tag{13}
$$

Equation (12) can be rearranged using equations (9) and (10) to show the relationship between sheet temperature and thickness :

$$
\frac{c}{(1-X)L}\cdot (T_s - \bar{T}) = \frac{H_s}{(1-X)L}\cdot \left(\frac{k_0}{k}\right)^{1/2} - 1.
$$
\n(14)



FIG. 2. Sketch defining the heat balance of the sheet.

*H.&G--3N* 

When the sheet is heated to the saturation temperature  $T = T_s$ , k reaches a maximum constant value  $k_{\text{max}}$ . Hence:

$$
\frac{H_s}{(1-X)L} = \left(\frac{k_{\text{max}}}{k_0}\right)^{1/2}.
$$
 (15)

Using this expression, equation (14) can also be written in the form:

$$
\frac{c}{H_s} \left(\frac{k_{\text{max}}}{k_0}\right)^{1/2} \cdot (T_s - \bar{T}) = \left(\frac{k_{\text{max}}}{k}\right)^{1/2} - 1. \quad (16)
$$

# The local heat flux

A heat balance over a differentiai element of the sheet (Fig. 2) shows that the latent and sensible heat released by the condensing steam on both sides of the sheet is given by:

$$
2q_x \cdot dA = \rho dQ [(1-X)L + c (T_s - \overline{T})] (17)
$$

Applying straight-forward manipulations, where  $q_x$  is local heat flux and  $dA$  the element area on one face of the sheet.

equation (17) can be transformed with the help of the previous equations to give the relationship between local heat flux and rate of thickness increase:

$$
q_x = 1/4 \cdot \rho \, H_s \, k_0 \, \bar{u}_0 \cdot \frac{1}{k_x} \cdot \frac{\mathrm{d}k}{\mathrm{d}x}.\tag{18}
$$

## *The local heat-transfer coeflcient*

The local heat-transfer coefficient is given by:

$$
h_x = \frac{q_x}{T_s - T}.
$$
 (19)

It should be noted that according to this definition,  $h_x$  is actually an overall coefficient of the system and not a "film" coefficient. Expressed in non-dimensional form, equation (19) becomes :

$$
Nu_x = \frac{2s \, h_x}{K} = \frac{2s \, q_x}{K(T_s - \overline{T})} \tag{20}
$$

where the Nusselt number is defined on the basis of the hydraulic diameter of the sheet.

the thickness variation, is found by combining equations (16), (18) and (20): situated at the radial distance  $x<sub>r</sub>$  from the

$$
\mathsf{V}u_x = \left[\frac{\rho c \,\tilde{u}_0 \, k_0}{2K} \cdot \left(\frac{k_{\text{max}}}{k_0}\right)^{1/2}\right] \cdot \left[\frac{1}{x^2} \cdot \frac{\mathrm{d}k}{\mathrm{d}x} \cdot \frac{1}{(k_{\text{max}}/k)^{1/2} - 1}\right] \tag{21}
$$

where the factors in the first brackets are constant while those in the second brackets are variables.

The local Craetz number corresponding to  $Nu_x$  is, by definition:

$$
Gz = \frac{\rho c \bar{u}(2s)^2}{K x} = \frac{4 \rho c \bar{u}_0(k_0)^{1/2} (k)^{3/2}}{K x^3}.
$$
 (22)

It is instructive to examine the experimental measurements required according to equations (21) and (22) in order to determine  $Nu_x = f(Gz)$ . The change of *k* with x is seen to be the only functional relationship between variables that need be established by experiment. It is not necessary to measure the temperature or the velocity variation along the sheet, Moreover, no information is needed either on steam temperature and quality, or on water inlet temperature, provided the sheet is heated to practical saturation so that  $k_{\text{max}}$  can be established. Physical properties appearing in the equations are, of course, temperature dependent. However, this variation is so slight that a rough estimate of the average temperature in the sheet is sufficient to determine their magnitudes.

#### *Interferometric determination of the thickness parameter*

When a monochromatic light beam is incident upon the surface of a fan spray sheet, interference lines can be observed by viewing the sheet in the direction of the reflection angle [S]. Each interference line represents the locus of equal thickness points. Figure 3 shows a schematic diagram of the fringe pattern produced. Since the thickness variation is at its maximum in the vicinity of the orifice, adjacent interference lines are very close to each other and cannot be resolved in that region. Hence, interferometric analysis of the thickness is restricted to the

region beyond that near the nozzle.<br>Let  $r$  denote the fringe number of the first The final expression, giving  $Nu_x$  in terms of Let r denote the fringe number of the first e thickness variation, is found by combining clearly resolved interference line, which is





FIG. 4. Interference photographs of a water sheet.



FIG. 3. Schematic diagram of the interference pattern.

origin. On this reference line, sheet thickness is  $s_r$ . If starting from this line,  $(n - r)$  fringes are counted when the distance  $x_n$  is reached, sheet thickness  $s_n$  at any point on the  $n<sup>th</sup>$  line differs from the original thickness according to [8]:

$$
s_r - s_n = B(n - r) \tag{23}
$$

*B* is a known optical constant varying only very slightly with temperature. It is given by:

$$
B=\frac{\lambda}{2\mu\left[1-(\sin\delta/\mu)^2\right]^{1/2}}\qquad(24)
$$

where  $\lambda$  is the wavelength of the monochromatic light,  $\mu$ —the refractive index and  $\delta$ —the angle of incidence.

Since the reference thickness  $s_r$  is unknown, the absolute thickness is not immediately available. However, there are sufficient data to determine the absolute thickness, provided the sheet exposed to steam practically reaches the saturation temperature in a zone still showing clear interference lines. This condition was met in all experiments of this investigation. For the thin sheets used, the temperature difference between steam and water became vanishingly small at a distance of about 2 cm from the nozzle, while interference lines could be observed up to a distance of 3 cm.

The following procedure describes the method of determining the desired thickness data appearing in the heat-transfer equations. Two interference photographs of the sheet under similar conditions are taken: one with the sheet in air surroundings, the other in an atmosphere of saturated steam (Fig. 4). Each photograph is analysed to give the variation of fringe number  $(n - r)$  with the reciprocal of  $x_n$ . A significant variable is the slope of the curve, which will be defined by:

$$
a = \frac{-d(n-r)}{d(1/x_n)}.\tag{25}
$$

Typical interferometric plots obtained in one of the tests are shown in Fig. 5.

The photograph of the unheated sheet taken in air surroundings is used to determine its constant thickness parameter  $k_0$ , which is also the value of *k* at the orifice for the steam heated sheet. Substituting for the thickness from equation (3), and noting that  $k_n = k_r = k_0$ , equation (23) can be rewritten as follows:

$$
\frac{1}{x_n} = -\frac{B}{k_0}(n-r) + \frac{1}{x_r}.
$$
 (26)

The slope  $a_0$  of the straight line obtained by plotting  $1/(x_n)$  versus  $(n - r)$  (Fig. 5) gives the value of *ko:* 

$$
k_0 = \frac{B}{a_0}.\tag{27}
$$

The general expression for determining *k*  as a function of x in the sheet with steam condensation is given by combining equations (5) and (23):

$$
\frac{1}{x_n} = -\frac{B}{k_n}(n-r) + \frac{1}{k_n} \cdot \frac{k_r}{x_r}.
$$
 (28)

Here, the plot of  $1/x_n$  versus  $(n - r)$  gives a curve of diminishing slope tending asymptotically to a straight line (Fig. 5). The region of



FIG. 5. Interferometric plots for the same sheet with and without steam condensation.

decreasing slope corresponds to the zone of Finally, substituting in equations  $(21)$  and  $(22)$  thickness increase due to condensation. The from equations  $(30)$  and  $(31)$ , the heat-transfer thickness increase due **to** condensation. The from equations (30) and (31), the heat-transfer final linear part of the curve represents the region where condensation is practicatly arrested. In the last region, *k* becomes constant having reached its maximum value  $k_{\text{max}}$ . This property can be used to substitute for the unknown reference thickness.

The value of  $k_{\text{max}}$  is obtained from the slope  $a_{\text{max}}$  of the linear part of the curve:

$$
k_{\max} = \frac{B}{a_{\max}}.\tag{29}
$$

Taking the reference thickness at a reliable point  $(n_{\text{max}} - r, 1/x_{\text{max}})$  of the straight line, equation (27) is transformed into:

$$
k_n = B\left\{\frac{x_n}{x_{\max} a_{\max}} + x_n[(n_{\max} - r) - (n - r)]\right\}
$$
\n(30)

Dropping the subscript  $n$  to denote that the continuous function may be used, the derivative of *k* is given by:

$$
\frac{dk}{dx} = B \left[ \frac{1}{x_{\text{max}} a_{\text{max}}} - \frac{1}{x a} + (n_{\text{max}} - r) - (n - r) \right]
$$
\n(21)

$$
Nu_x = \frac{\frac{F}{2a_0} \cdot \frac{1}{x^2} \left[ \frac{1}{x_{\text{max}}} - \frac{1}{x a} + (n_{\text{max}} - n) \right]}{\left[ \frac{x}{x_{\text{max}}} + a_{\text{max}} x (n_{\text{max}} - n) \right]^{-1/2} - 1}
$$
\n(32)  
\n
$$
4F \cdot \frac{a_{\text{max}}}{a_0} \cdot \frac{1}{x} \left[ \frac{1}{x_{\text{max}} a_{\text{max}}} + (n_{\text{max}} - n) \right]^2
$$

$$
\max_{\text{max}} = \frac{a_0}{a_{\text{max}}} \qquad (29) \qquad G_Z = \frac{a_0 \left[ \frac{x}{a_{\text{max}}} \right]^{4 \text{ max} a_{\text{max}}} \left[ \frac{a_{\text{max}}}{x_{\text{max}}} \right]^{1/2}} \qquad (33)
$$
\nsince thickness at a reliable

\n
$$
1/x_{\text{max}} \qquad \text{of the straight line,} \qquad (33)
$$

where the constant  $F$  is given by:

$$
F = \frac{\rho c \bar{u}_0 B^2}{K} \cdot \left(\frac{a_0}{a_{\text{max}}}\right)^{1/2}.
$$
 (34)

Thus, values of  $Nu_x$  as a function of  $Gz$ along the sheet, can be established by taking slopes at several points on the curved part of the graph of  $(1/x_n)$  versus  $(n - r)$ .

#### **EXPERIMENTAL SYSTEM**

The essential parts of the experimental (31) system (Fig. 6) were as follows:



FIG. 6. Layout of experimental system.

*Steam pressure vessel.* Steam condensation occurred on a spray sheet flowing inside an insulated steel pressure vessel 3 ft long, 16 in diameter. The vessel was equipped with four Pyrex glass observation ports (5 in diameter,  $\frac{3}{4}$  in thick) allowing illumination and photography of the sheet. Silicon rubber 0 rings, withstanding the steam temperature, were used to seal the glass plates. The vessel was connected to a steam supply and a vacuum pump. It was operated at steam pressures in the range of 10 psig to 24 in Hg vacuum. A lower vacuum could not be reached due to the high vapour load on the pump.

Nozzle *holder.* The spray nozzle was vertically

held at the top of the vessel by means of a holder whose details are shown in Fig. 7. A stuffing box arrangement allowed rotation of the nozzle holder so as to regulate the position of the sheet with respect to light source and camera.

The holder was designed to minimize heating in the feed channel leading to the nozzle. The feed channel was drilled in a thick Teflon rod providing thermal insulation to the flowing water. A pressure tap and a tap for the temperature measuring thermistor were drilled at the minimum practicable distance above the nozzle. Particular care was taken to ensure insulation of the water channel between the thermistor connexion and the nozzle.



**FIG.** *7.* Nozzle holder.

*Water collector.* To prevent fogging of the vessel by atomization, the unbroken sheet was drained through a collector. This consisted of a horizontal Ebonite pipe sealed at its extremities. A thin slot at the top allowed passage of the sheet and a drain at the bottom led the collected water to an outlet of the vessel.

*Spray nozzles.* The spray nozzles used were Bray Miniature Unijets manufactured by Geo. Bray and Co. Ltd., Leeds, England. Most of the experiments were carried out with nozzles designated as X and Y by the manufacturers. The dimensions of the rectangular orifice, measured by microscope, are given in Table 2. The orifice area was determined by graphical integration of the orifice image, taking into account the slight irregularities of the edges.

*Table* 2. *Dimensions of nozzles* 

	Nozzle Orifice length (cm)	Orifice breadth Orifice projec- (cm)	ted area $(cm2)$	
x	0.7043	0.3384	$2.372 \times 10^{-3}$	
Y	0.5573	0.3201	$1.735 \times 10^{-3}$	

A fine mesh screen, fixed near the nozzle inlet, proved effective in preventing sheet vibrations. These vibrations blur the interference pattern to the naked eye but not to the photographic image taken at an exposure time of  $\frac{1}{10}$  to  $\frac{1}{25}$  s.

*Feed vessel.* Soft water at room temperature was stored in a feed vessel. Compressed air at a pressure of 75 to 90 psig in the feed vessel, was used to push the water through the spray nozzle. The injection pressure at the nozzle was finely regulated in the range of 8 to 30 psi pressure differential, by means of a reducing valve.

This arrangement was varied in only one series of tests. In the experiments verifying theoretical predictions assuming no surface resistance, de-aerated water was used. This was obtained by bubbling steam through water kept at its boiling temperature. The hot de-aerated water was injected through the nozzle by steam pressure. Before reaching the nozzle, the water

was brought down to room temperature by the cooler shown in dotted lines in Fig. 6.

*Measuring instruments.* Throughput of the water issuing from the nozzle was held constant by controlling the injection pressure. Water velocity at the orifice exit was obtained from the measured flow rate and the orifice projected area, assuming that the coefficient of contraction is unity [8].

The thermistor fixed close to the nozzle gave the inlet water temperature. The temperature was obtained from the thermistor resistance measured by a null Wheatstone bridge. The thermistor leads were protected from steam by passing them through a rubber hose issuing out of the vessel. Air blown through the hose prevented the leads from overheating.

Though accurate temperature measurement was not essential for the main tests, it was necessary for the initial experiments checking the heat balance on the sheet.

The pressure in the steam vessel was measured to a sufficient degree of accuracy by a Bourdon gauge, at above atmospheric pressures, and by a mercury U tube manometer, at sub-atmospheric pressures. A bimetal thermometer gave the steam temperature in the vessel.

*Sheet illumination and photography.* The layout of the photographic system is shown in Fig. 8. The sheet was illuminated by a monochromatic light beam (wavelengths 5890 and 5896 A) obtained from a 45 W sodium lamp. The position of the sheet was fixed such that directly reflected light reached the camera.

The sodium light source did not provide sufficient illumination to show the nozzle body.



**FIG.** 8. Photographic layout.

It was essential to have the nozzle body appear on the photographic plate, in order to fix the origin for the distance co-ordinate  $x$ . This was achieved by additional illumination from a photoflood lamp, throwing light in the parallel direction of the sheet, so as not to disturb the interference pattern. Typical photographs obtained are shown in Fig. 4.

*Analysis of the air content qf the steam.* Air content was determined by separating and collecting the air mixed in a measured weight of steam. A gas sample was sucked from the steam vessel into the glass analysing apparatus (Fig. 9) from a point close to the nozzle, giving an air concentration representative of the steam. Pure steam was separated from the air component by adsorption on a bed of molecular sieves (type 4A) held in a U tube. A similar U tube, connected in series with the first one, served , to check that all of the steam was adsorbed on the first bed. Steam free air, passing the U tubes, was collected in a spherical glass bulb of known volume. The bulb was connected to a mercury manometer and to a thermometer. The pressure rise in the bulb and the weight of adsorbed steam enabled calculation of the air concentration.

Low pressure of the system was an essential requirement for reducing an error arising from the heat of adsorption, as explained below. The analysing apparatus was therefore initially evacuated, by means of a high vacuum pump, to an absolute pressure of less than 1 mm Hg absolute. Air concentration was calculated assuming that the temperature is equalized throughout the system when the sampling operation is terminated. According to ideal gas laws, air concentration,  $C$  per cent by volume, is given by:

$$
C=\frac{\Delta P}{RT_1}\frac{(V_b+v)}{W}\times 18\times 100\qquad \quad (35)
$$

where  $\Delta P = P_3 - P_1$  is the air pressure rise recorded by the manometer,  $W$ —the weight of steam captured on the adsorbent bed,  $V_b$ —the volume of the bulb, v—the volume of other voids in the system,  $T_1$ —the absolute temperature read from the bulb thermometer, and *—the gas constant.* 

Actually the assumption that air temperature is equalized in the system can lead into a major error. The molecular sieves were considerably heated during the adsorption process and a long time was necessary for temperature equalization. The provision of an effective cooling system would have complicated the apparatus. Hence, no effort was made to remove the heat of adsorption and the pressure difference  $\Delta P$  was read soon after sampling. Hot air,



*DIMENSIONS IIf MM* 

FIG. 9. Apparatus for analysing air content of steam.

trapped over the adsorbent bed, was then at a higher temperature,  $T_3$ , which was not exactly known. Part of the pressure rise recorded by the manometer was due to a temperature difference between the hot air over the adsorbent material and the air in the bulb. The pressure difference,  $\Delta P_1$ , due to the heating effect, represents an error in  $\Delta P$ . This error can be calculated from the approximate equation :

$$
\Delta P_1 = \frac{v}{V_b} \cdot \left(1 - \frac{T_1}{T_3}\right) \cdot P_3 \tag{36}
$$

where  $P_3$  is the absolute pressure in the analysing apparatus immediately after sampling, and temperatures are expressed in an absolute scale.

The trapped volume was estimated at  $v =$ 20 cm<sup>3</sup>, and its maximum temperature, at less than 150°C. The air collected in the bulb was at a temperature around 25°C. Bulb volume was  $V_b = 760$  cm<sup>3</sup>. Substituting these values in equation (36), the percentage relative error  $e$  is given by:

$$
e = \frac{\Delta P_1}{\Delta P} \times 100 = 0.8 \left( 1 + \frac{P_1}{\Delta P} \right). \tag{37}
$$

Depending on air concentration, values of  $\Delta P$  ranged from 1 to 50 mm Hg. These values could not be safely increased further by taking a larger sample size because of the danger of steam breakthrough into the air bulb.

Had the pressure of the system  $(P_1)$  been around atmospheric, the percentage error, as shown by equation  $(37)$ , would have amounted to several hundred per cent at the lower air concentrations. Since the error is in direct proportion to the pressure of the system, sampling was carried out at a low pressure. With *PI less* than 1 mm Hg absolute, the error was less than 2 per cent for all air concentrations.

Repeat analyses gave results in agreement with each other within 10 per cent. **INITIAL RESULTS** 

*Experimental procedure.* A test consisted of taking two interferometric photographs of the same sheet, one with the vessel open to the atmosphere and the other with steam in the vessel. The operating procedure for the latter case was as follows. The vessel was filled with steam mixed, if desired, with air in suitable cent by volume.

proportions. To prevent air accumulation in the vessel and to avoid fog formation obscuring the sheet, a small amount of steam was continuously bled out of the vessel when at a pressure above atmospheric. In sub-atmospheric steam tests, suction of the vacuum pump provided the necessary purging action.

It should be noted that the velocity of the steam due to the bleeding off was insignificant. The bulk of the steam was virtually stagnant in view of the large cross-sectional area of the vessel. Several repeat photographs of the sheet were taken when steady state was reached. Air concentration in the steam was measured before and after taking the photographs.

Measurement of the interferometric data from the photographic plate were made on a projected image of the negative, magnified by an enlarger to exactly  $\times$  10 the natural size. The accuracy in measuring the distance  $x$  was about 0.03 mm.

The interferometric count was carried for one or two radial lines along the centre of the sheet (Fig. 4). lnterferometric measurements usually covered the range of 0.5 to 3 cm. from the nozzle.

The origin of the  $x$  co-ordinates is slightly behind the nozzle exit (Fig. 1). Accuracy of the measurements was improved by taking into account the interval between the nozzle exit and the hypothetical origin of the sheet. The magnitude of this interval (0.5 mm) was estimated from the angle of the tapered walls guiding the sheet issuing from the orifice and from the dimensions of the nozzle exit.

The rate of thickness increase due to steam condensation, detected by the measurements, was of the order of 1 micron per cm sheet length. This compared with a rate of thickness decrease due to thinning out of the sheet of about 5-10 micron per cm.

The initial tests served to check the reliability of the experimental method and to examine its scope. These tests were carried out with practically pure steam at above atmospheric pressures (2 to 10 psig). Air content of the steam, roughly measured in the initial tests, was O-02 to 0.04 per

# *Check of the heat balance on the sheet*

The interferometric determination of the local heat-transfer coefficient is based on the assumption that the velocity profile in the transverse direction is the same, with and without steam condensation [equation  $(10)$ ]. A full check of this assumption, by an independent heat balance, would be available if it were possible to obtain an accurate measurement of either velocity, flow rate or temperature at some point on the steam heated sheet.

Efforts were made to determine the flow rate and temperature of the sheet at the point of its inlet into a specially designed collector. However, it was found impossible to realize an independent heat balance with a degree of accuracy sufficient to detect a significant difference from the precise balance given by the interferometric data.

An indirect check of the velocity profile assumption was made by examining the interferometric heat balance for saturation conditions. If the velocity profile term is taken into account, equation (15), giving a heat balance based on thickness measurements, takes the form :

$$
\frac{H_s}{(1-X)L} = \gamma_{\text{max}} \left(\frac{k_{\text{max}}}{k_0}\right)^{1/2} = \gamma_{\text{max}} \left(\frac{a_0}{a_{\text{max}}}\right)^{1/2} \tag{38}
$$

where  $\gamma_{\text{max}}$  is the ratio of velocity means defined by equation (8), at the saturation point.

The thickness parameters  $k_{\text{max}}$  and  $k_0$  were accurately known from the straight line slopes *ao* and *amax.* The temperature of the saturated steam was also accurately known. The wetness fraction of the steam entering the vessel was measured by a throttling calorimeter and found to be negligibly small  $(X = 0.01 - 0.02)$ . It is reasonable to expect that the steam condensing on the sheet is slightly more wet. Hence, if the velocity profile remains constant, values of  $\gamma_{\text{max}}$  calculated from equation (38) with  $X = 0$ , are expected to be slightly less than 1.

This prediction was confirmed by the experimental results. Table 3 shows data obtained with steam at 3 psig. Values of  $\gamma_{\text{max}}$  are seen to fall consistently in the vicinity of 1, most of them being below 1. Random experimental error accounts for the few values of  $\gamma_{\text{max}}$  which are above 1.

Nozzle	$\bar{u}_0$ (cm/s)	$\gamma$ max
	1014	0.9930
	1014	0.9765
	1014	0.9972
	1354	0.9954
Y	1354	0.9888
	1354	0.9897
	1354	1.007
	1700	0.9847
	1700	0.9788
	1189	0.9590
	1189	0.9416
	1309	0.9637
x	1309	0.9810
	1530	1.019
	1530	1.013
	1619	0.9999
	1619	1.0068

*Table 3. Values of the velocity means ratio measured at saturation* 

*Limits of the Graetz and Nusselt numbers* 

Determination of the local heat-transfer coefficient was confined to a narrow range of low Graetz numbers, dictated by experimental considerations. Because of the difficulty of resolving the dense interference fringes in the upper part of the sheet (Fig. 4), measurements of the heat-transfer coefficient were obtained only for the lower part of the sheet  $(x$  approximately  $0.5$  to  $2.5$  cm and s, 5 to 20 micron). The minimum water velocity giving a sufficiently extended sheet was around 800 cm/s. An upper limit to the velocity (about 1600 cm/s) was imposed by the need to prevent early sheet waviness which obliterates the interference pattern.

The Graetz number range corresponding to the above limits is 1 to 12. For this range of low Graetz numbers, theory [2] predicts that  $Nu_x$ should have the same constant value for all values of the Graetz number. This prediction was confirmed by the experimental results. The Nusselt number was found to remain constant within  $+35$  per cent, irrespective of nozzle size, sheet thickness or water velocity. Table 4 gives typical results, obtained in two replicate experiments.

$x$ (cm)	Gz	$N u_r$	$x$ (cm)	Gz	$N u_r$
$1 - 0.3$	$10-4$	4.8	0.94	12.6	5.4
$1-10$	8.3	3.7	1.025	9.9	4.7
$1 - 21$	6.4	$3 - 7$	$1 - 12$	7.7	4-4
1.315	5.0	$3-4$	$1 - 24$	5.8	3.3
1.44	3.9	40	and the state of the state of the		
and the state of the state				<b>Contract Contract</b> Average: $Nu_x = 4.5$	<b>A R &amp; H &amp; COMPOSITION</b>

Table 4. Typical variations in the value of Nu<sub>x</sub> as shown *by two replicate experiments* 

A random scatter of  $\pm 35$  per cent is not unusual in measurement of the local heattransfer coefficient. Any method for determining this coefficient suffers from poor accuracy since it requires evaluation of the small difference between two large quantities.

Knowing that  $\overline{Nu}_x$  is a constant, it was possible to establish a confident estimate of its value by taking the average of all measurements. Analysis of more than 30 photographs of the sheet gave a mean value of  $Nu<sub>x</sub>$  of about 5. This is only half the theoretical value of 9.87, predicted from the solution for the fan spray sheet with no surface resistance [2].

The discrepancy between the theoretical and experimental value of  $Nu_x$  could represent a difference arising from unjustified simplification of the theoretical model. Alternatively, it could be due to the existence of a surface resistance on the sheet. In the latter event, the measured value of  $Nu_x$  should approach the theoretical one when surface resistance is removed.

# CONDENSATION ON A DE-AERATED WATER SHEET

Efforts were made to find experimental conditions ensuring the absence of surface resistance on the sheet. It was initially thought that inert gases, accumulating somehow in the system, were responsible for the occurrence of a resistance on the sheet. Measures were therefore taken to eliminate this possibility by vigorous venting of the steam supply source and the experimental vessel.

Careful analysis of the air content in the system showed that so-called pure steam, inevitably contains a small quantity of inert gases (about  $0.015$  to  $0.02$  per cent), a result also found by a previous worker [6]. Repetition of some of the tests under carefully controlled conditions gave essentially the same value of  $Nu<sub>x</sub>$  as obtained in the initial tests.

It was finally discovered that the appearance of surface resistance in the condensation of practically pure steam is caused by the release of dissolved air from the heated water sheet. Evidence for this effect was found in six replicated tests carried out with de-aerated water. Nozzle Y was used and water velocity was 1460 cm/s. The pressure vessel was at a steam pressure of 2 to 3 psi.g The de-aerated water was injected into the pressure vessel using steam pressure in the feed vessel, instead of compressed air employed in all other tests (Fig. 6).

Table 5 summarizes the results obtained with de-aerated water. It is seen that  $Nu_x$  has a value approaching the theoretical one. The average  $Nu_x$ , based on all the measurements of the runs in Table 5, is 9.2. This result is in good agreement, within the limits of experimental accuracy, with the theoretical value of 9.87 predicted by the analysis of the fan spray sheet with no surface resistance. This experimental confirmation provides considerable confidence as to the applicability of the theoretical solutions for other jet configurations, derived elsewhere [2].

*Table 5. Nusselt numbers mearured in rhe condensation of practically pure steam on de-aerated water.* 

Run	Graetz number		Nusselt number			
no.				Minimum Maximum Minimum Maximum Average		
	4.2	$10-8$	65	11.0	$8-2$	
2	$5-2$	$12-3$	8.2	$11-5$	9.7	
3	4·1	8.4	$6-4$	8.5	7.3	
	5.3	12.2	9.2	11-0	9.9	
5	40	6.6	$10-4$	$13-1$	$12-1$	
6	7.9	10.2	6.1	8.6	8·1	

In the sheets investigated here, the released air layer offered a thermal resistance of the same order of magnitude as the conduction resistance of the water. Although the dissolved air content of water is small (about 100 cm3/1 at NTP under conditions existing in the feed vessel), this air can only be discarded from the sheet surface by diffusing. against the flow of the condensing steam. It should be possible to reduce the resistance due to dissolved air by forcing the steam to flow in the direction of the sheet, so as to enhance air removal from the sheet surface.

# CONDENSATION IN VACUUM AND EFFECTS OF AIR IN STEAM

The final and main experiments were aimed to find whether the heat-transfer coefficient is significantly reduced in vacuum due to the interface resistance effect and to obtain quantitative data on the variation of the coefficient with increase of air content in steam. The tests were carried out with one nozzle (Y) and at a constant nominal water velocity (1430 to 1470 cm/s). To simulate the more practical conditions, water was injected from the feed vessel by air pressure.

Two series of tests were performed with air concentration in the steam increasing in each case up to 2.7 per cent by volume. In the first series, steam in the pressure vessel was at a nominal vacuum of 24 in Hg, and in the second series, at a pressure of 1.8 psig. Values of the measured overall local heat-transfer coefficient, expressed by the Nusselt number, are given in Table 6.

In general, the resistance due to air in steam should have been evaluated from the variation of the local heat-transfer coefficient with air concentration, at points of equal sheet thickness. This method of analysis was considered impractical, since further experimental scatter would have been introduced due to the variation of  $h<sub>x</sub>$ with sheet thickness. The effect of air concentration on heat transfer was investigated with better confidence by taking  $Nu<sub>x</sub>$ , instead of  $h_x$ .

The experimental data showed that  $Nu<sub>x</sub>$ remained constant along each sheet, within the accuracy limits of the measurement method (Table 6). This result finds support in the

steam pressure. (mm Hg) 133 133 142 142 133 133 146	Air content $(C\%)$ 0.18 0.18 0.61 0.61 0.71 0.71	4.3 5.3 $3-0$ 3.0 $4-1$ 2.3	Minimum Maximum Minimum Maximum 6.3 $6-0$ 7.1 4.0 $6 - 4$	4.5 4.3 $3-1$ 3.2	5.9 5.6 3.7 4.0	Average $5-2$ $5-0$ $3-4$ 3.6
				$4-0$	$6-2$	5.3
			$6-1$	$2-0$	$3-1$	2.4
	1.54	1.5	$5-3$	$1-0$	1.9	$1-2$
142	2.31	$2 - 1$	4.7	0.7	1.8	$1-2$
148	2.66	$1-3$		0.6		0.8
148	2.66	0.6	$5-7$	0.5	$1-0$	0.7
850	0.02	4.4	6.7	5.8	6.8	6.4
850	0.02	4.7	9.3	4.4		4.9
850	0.50	2.6	5.0			2.9
850	0.50	$2 \cdot 1$	7·0	2.2	2.7	2.4
850	0.93	1.6	9.5	2.2		2.5
850	0.93	2.8	5.4	1.7	20	1.9
850	1.59	1.9	$6-3$	$1-7$	$2-3$	$2-0$
850	1.59	20	9.2	1.9	2.5	$2 - 1$
850	2.65	$2-0$	$8-7$	$1-2$	1.9	$1-4$
850	2.65	$2 - 0$	$10-0$	$1-2$	$2 - 0$	$1-6$
				$5-4$	2.5	0.9 5.2 $3-4$ 3.0

Table 6. Values of Gz and  $Nu_x$  as a function of air concentration in steam at two levels *of the steam pressure* 



FIG. IO. Variation of the Nusselt number with air content of steam at two levels of steam pressure.

analytical solution for a plane sheet of uniform thickness with a constant external resistance, derived elsewhere [2]. If it is assumed that for the narrow range of sheet thicknesses investigated the fan spray sheet may be approximated by the uniformly thick sheet,  $Nu_x$  is expected to remain constant in the range of low Graetz numbers covered by the measurements  $(Gz)$ 1 to 10).

Figure 10 shows the variation of the average value of  $Nu_x$  of each sheet, with air content in steam at the two levels of steam pressure. An important conclusion is that, for a given air concentration, there is no significant difference between values of  $Nu_x$  at sub-atmospheric and at above atmospheric steam pressure.

The decrease in  $Nu_x$  with increasing air concentration is rapid at low air concentrations, becoming slow at the relatively high air concentrations. The relationship between  $Nu<sub>x</sub>$  and C appears to be exponential in form, as shown by the straight line representing the data on semi-logarithmic co-ordinates (Fig. 10). The empirical equation relating  $Nu_x$  with air concentration, found by linear regression, is:

$$
Nu_x = 4.8 \exp(-0.60 \text{ C}) \tag{39}
$$

where  $C$ , per cent by volume air, is within the limits  $0.015 < C < 2.7$ .

It is of interest to compare the relative decrease in the heat-transfer coefficient due to air in steam as found for condensation on water sheets with that for condensation on tubes.

Figure 11 shows the ratio  $h/h_0$  of the heattransfer coefficient with air in the steam relative to the value obtained with practically pure steam as a function of air concentration, for results of this work and for condensation on tubes. The curve for the latter case was derived by Kutateladze [7] who correlated the results of several workers. Despite the many differences between the two steam condensation systems, the decrease in the relative heat-transfer coefficient is almost identical in the two cases.



FIG. II. Ratio of the heat-transfer coefficient with air in steam to the coefficient with practically pure steam as a function of air content.

# INTERFACE RESISTANCE IN CONDENSATION

Some conclusions regarding the magnitude of the condensation coefficient may be derived from the observation that the value of the heattransfer coefficient is not significantly altered

by the decrease of steam pressure. With practically pure steam, the same value of  $Nu_x$  (approximately 4.8) was obtained in condensation occurring both at a pressure slightly above atmospheric and at a vacuum of about 25 in Hg. Sheet thickness in the measurement region varied between 7 to 12 micron, showing that the overall local heat-transfer coefficient was of the order of 200 000 to 100 000 kcal/h m2 degC.

Had the magnitude of the condensation coefficient, a, been 0.036, the interface heattransfer coefficient, in steam condensation at a vacuum of 25 in Hg, would have amounted to  $65000$  kcal/h m<sup>2</sup> degC (Table 1).

However, it is impossible for this value of an individual coefficient to be lower than that of the overall coefficient. Hence, it is evident that, under the conditions of the present investigation, interface resistance was negligible and that the magnitude of  $\alpha$  was considerably higher than O-036. In the light of this experimental finding, it appears doubtful to include interface resistance in calculating the overall coefficient of vacuum condensers, as has been suggested by Silver [3].

In a previous investigation of methanol vapour condensation on a solid surface in the presence of air, Baer and McKelvey [4] interpreted their results assuming that  $\alpha$  varies with pressure and air concentration. If this view, ascribing all surface resistance to the interface heat-transfer coefficient, is accepted, the results of the present work imply that  $\alpha$  increases with decreasing pressure. The reason is that the sum of the conduction and interface resistances does not vary with pressure while interface resistance increases with decreasing pressure (Table 1). An opposite trend for the variation of **a** with pressure was observed by Baer and McKelvey.

There seems to be little point in expressing heat-transfer resistance due to pressure or air content effect through values of  $\alpha$ . As yet, there is no theoretical guidance on the magnitude of  $\alpha$ or even the direction of its change with pressure. If **a** is not considered constant, it loses its empirical value and results are better presented through the heat-transfer coefficient.

# **DESIGN EQUATION**

This experimental investigation was confined to the low range of Graetz numbers for which

 $Nu_x$  is a constant, depending on air concentration in steam. For larger Graetz numbers, theory [2] predicts that  $Nu_x$  increases with Gz. However, the Graetz number decreases rapidly with the distance from the origin, since  $Gz \approx (1/x^3)$ , and it may be taken that  $Nu_x$  reaches high values only in a narrow region of the sheet.

Assuming that  $Nu_x$  is constant over the whole extent of the sheet, a simple design equation containing a margin of safety may be developed for estimating water temperature as a function of x. The basic heat-transfer equation is:

$$
\rho \ Q \ c \ d\bar{T} = 2h_x \ dA \ (T_s - \bar{T}). \tag{40}
$$

Using equations  $(4)$ ,  $(9)$  and  $(14)$ , equation  $(40)$ is transformed into :

$$
\int_{T_0}^{T} \frac{1}{T_s - T} \cdot \left[ \frac{c (T_s - T_0) + L}{c (T_s - T) + L} \right]^3 dT
$$
\n
$$
= \frac{Nu_x K}{\rho c u_0 k_0^2} \int_{0}^{x} x^2 dx \quad (41)
$$

Equation (41) can be integrated analytically, but the expression obtained is rather cumbersome. Further simplification may be made by noting that the term in brackets in the temperature integral varies slowly with  $\bar{T}$ , tending to a maximum, *(Hs/L)3.* Substituting this maximum value for the varying term, the approximation made is in the safe direction. Integration of equation (41) then yields:

$$
\frac{\overline{T} - T_s}{T_0 - T_s} = \exp\left[ -\frac{4}{3} \cdot \left(\frac{L}{H_s}\right)^3 \cdot \frac{Nu_x}{Gz_0} \right] \quad (42)
$$

where the Graetz number obtained here is based on known exit conditions of velocity and thickness parameter:

$$
Gz_0 = \frac{4 \rho \bar{u}_0 k_0^2}{K x^3} \tag{43}
$$

Depending on air content of the steam, the  $Nu_x$ appearing in equation (42) can be estimated from equation (39).

Equations (42) and (43) were specifically derived for the fan spray sheet. However, in the absence of other data, they can be used to estimate the temperature rise for other types of sheets (e.g. the swirl spray sheet) by substituting for  $k_0$  the appropriate thickness parameter.

In conclusion, it may be noted that this study, confined to laminar jets, provides also guidance for calculations involving turbulent jets. Any temperature estimate for a turbulent jet made under the assumption of laminar flow, gives a design value containing a safety margin.

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**Résumé--On a développé une méthode pour l'étude du transport de chaleur dans la condensation de** vapeur sur des lames d'eau laminaires produites par un ajutage donnant un jet en éventail. Les coefficients locaux de transport de chaleur sont déterminés d'une manière sûre, sans instruments de sondage, à partir de mesures interférométriques de l'augmentation d'épaisseur due à la condensation de la vapeur.

On trouve que les faibles quantités d'air dissous, qui se dégagent durant le chauffage du jet plan augmentent considérablement la résistance au transport de chaleur. On a fait des essais avec de l'eau désaérée pour confirmer la solution analytique du chauffage du jet plan en éventail sans résistance de surface, qui a été présentée dans un article précédent [2].

Les effets de la pression de la vapeur et de sa teneur en air sur ie coefficient de transport de chaleur sont examinés. On trouve que le coefficient de transport de chaleur, quoique très élevé (100 000  $\dot{a}$ )  $200\,000$  Kcal/h m<sup>2</sup> degC), reste inchangé lorsque la pression de la vapeur est réduite d'une valeur supérieure à la pression atmosphèrique à une valeur inférieure. Ce résultat implique que la résistance à l'interface est négligeable dans un équipement pour la condensation dans le vide, contrairement à l'opinion exprimée par quelques auteurs.

La diminution du coefficient de transport de chaleur avec I'augmentation de la teneur en air de la vapeur est corrélée empiriquement. On obtient une équation simple de prédiction qui, en même temps que la corrélation empirique, peut être employée pour des calculs pratiques de l'équipement pour la condensation par contact direct.

**Zusammenfassung—Es** wurde eine Methode entwickelt, den Wärmeübergang bei der Kondensation von Wasserdampf and laminaren, von einer Fächersprühdüse erzeugten Wasserschichten zu untersuchen. Örtliche Wärmeübergangskoeffizienten lassen sich zuverlässig aus interferometrischen Messungen der Dickezunahme durch Dampfkondensation bestimmen.

Es zeigt sich, dass geringe Mengen gelöster Luft, die bei der Beheizung der Schicht frei werden, den Wärmeübergangswiderstand beträchtlich erhöhen. Versuche mit luftfreiem Wasser bestätigen die analytische Lösung für die Beheizung der fächerförmigen Sprühschicht ohne Übergangswiderstand, die in einer vorhergehenden Arbeit [2] wiedergegeben wurde.

Die Einflüsse des Dampfdruckes und des Luftgehaltes des Dampfes auf den Wärmeübergangskoeffizienten werden untersucht. Es zeigt sich, dass der sehr grosse Wärmeübergangskoeffizient (100 000 bis 200 000 kcal/h m<sup>2</sup> grd) bei einer Senkung des Dampfdrucks von Werten grösser als 1 Atmosphäre auf solche darunter, unverändert bleibt. Dieses Ergebnis bedeutet, dass der Übergangswiderstand bei der Kondensation im Vakuum vernachlässigbar ist, im Gegensatz zur Ansicht einiger Autoren.

Der Abfall des Wärmeübergangskoeffizienten bei zunehmendem Luftgehalt im Dampf wird empirisch korreliert. Eine einfache Gebrauchsgleichung wird abgeleitet; sie erlaubt, zusammen mit der empirischen Korrelation praktische Berechnungen ftir Kondensationsanordnungen mit Direktberiihrung.

Аннотация--Paзpaботан метод исследования теплообмена при конденсации пара на ламинарной водяной веерной струе, выходящей из форсунки. Локальные коэффициенты теплообмена определяются надежно без специальных зондов по интерферометрическим измерениям увеличения толщины струи в процессе конденсации пара.

Обнаружено, что небольшое количество растворенного воздуха, выделяющегося при нагревании струи, значительно повышает сопротивление переносу тепла. Опыты проводились с деаэрированной водой для подтверждения приведенного в работе [2] аналитического решения нагрева веерной струи при отсутствии поверхностного со-**JlIpOTHBJIeHMH.** 

Исследовалось влияние давления пара и содержания воздуха в паре на коэффициент теплообмена. Обнаружено, что очень высокий коэффициент теплообмена (от 100000 до 200000 ккал/час м<sup>2</sup> C<sup>o</sup>) не меняется при изменении давления пара в диапазоне значений от выше атмосферного до ниже атмосферного. Это означает, что в вакуумной конденсационной установке сопротивление поверхности раздела пренебрежимо мало в противоположность взгляду, которого придерживаются некоторые авторы.

Предложены эмпирические соотношения, связывающие уменьшение коэффициента теплообмена с увеличением содержания воздуха в паре. Выведено простое расчетное уравнение, которое вместе с эмпирическим соотношением можно использовать лля практических расчетов конденсационной установки непосредственного контакта.